Table 1 Required mirror widths for various nadir angles N and sensor beamwidth angles α

M, deg	N, deg	B, deg	B, miles	Required mirror width, in. $\alpha = 0.2°$	$0.4°$	$0.6°$
0.1	0.2	0.1	6.9	21.5	28.0	31.6
0.25	0.5	0.2	13.8	12.3	18.4	23.2
0.5	1.0	0.4	27.6	8.5	12.0	15.8
1	2	0.9	62	3.8	7.0	9.7
2	4	...	...	2.0	3.8	5.5
4	8	...	...	1.0	2.0	2.9
10	20	...	...	0.4	0.8	1.2
20	40	...	...	0.2	0.4	0.6

other suitable and telemetered reference is placed exactly midway between the centers of two helical mirror pairs, the time of reference telemeter could be used as a check on satellite and mirror attitude, since the center of each earth scan (as determined by the horizon crossings) should be displaced $n\pi/4$ (where $n = \pm 1, 3, 5 \ldots$) radians of satellite rotation from the reference.

The foregoing error analysis has been based on a constant rate-of-turn helical mirror whereby M is proportional to P. From the viewpoint of data processing and display, a more elegant configuration would have P proportional to B ($B = KP$, where K would be determined by the length of satellite circumferential arc used for the mirror) over the swath of normally meteorologically useful data—say, out to 500–700 miles from the subpoint track. This can be accomplished by a helical mirror configuration such that

$$M = \frac{1}{2}\cot^{-1}\left[\frac{R+h}{R\sin(KP)} - \cot(KP)\right]$$

within this range. Beyond the range of meteorologically useful data, a linear rate of change, $M = K'P$, could be resumed, with K' being a comparatively large constant to insure horizon-crossing detection without use of any unduly long helical mirror.

Another matter to be considered is the required width of the mirrors, which obviously must be larger for small values of M. Data on this point are given in Table 1 for several angles of sensor beamwidth α in the event that grosser linear resolution was deemed adequate or advisable. As might be expected, unduly large mirror widths would be required only for small values of M and N, say $M < 1°$. Here, because of the very small angles of both M and N, much of mirror width could be within the satellite's outer circumference in the optical path segment. For $M \leqslant 0.25°$, where even this might be insufficient, omission of such mirror angles would leave only a negligible gap (≤ 25 miles) between the vertical reading at the subpoint and the remainder of the cross-track scan.

In summary, the helical mirror configuration for cross-track scanning appears to be feasible and worthy of further investigation if cartwheel satellites are employed for meteorological purposes.

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Landing Site Coverage for Orbital Lifting Re-Entry Vehicles

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THIS note presents some results that augment two recently published articles concerning the recall of lifting re-entry vehicles.[1, 2] References 1 and 2 generally determine the cross-range maneuver required when the recovery areas are located in an optimum fashion with respect to a particular orbit. The analytical approach presented here provides a means for evaluating the return capability to specific landing sites whose latitudes are not necessarily unique functions of the orbital inclination. The technique is therefore applicable to cases where it may not be practical to provide optimal orbital inclination/landing-site latitude orientation.

The analysis involves the determination of the loci of points along the latitude of a specific landing site at which an orbiting vehicle with cross-range capability could land within an orbit period. The loci of such points form a belt or belts along the landing site latitude; the magnitudes of these arc lengths are defined as "latitude coverages" and will be referred to as such in further discussion. The concept of latitude coverage is of particular usefulness in the classes of missions in which the orientation of the vehicle's ascending node and the in-track position are random or arbitrary at the time recall is initiated. Such situations will generally exist when recall is initiated after relatively long-duration space missions, or when the selection of orbit altitude and inclination is imposed by operational considerations. The magnitude of the latitude coverage facilitates the determination of 1) the cross-range required to assure that a landing opportunity would exist with a specific time period, and 2) the probability of achieving a landing opportunity at a specific landing site within a specific time period.

The latitude coverage or loci of points along the latitude of a landing site at which an orbiting vehicle could land during one orbit period will occur in two general modes, depending upon the relation of the orbital inclination, landing site latitude, and the magnitude of the cross-range capability. The first mode is depicted in Fig. 1a in which one continuous latitude coverage belt is generated. A continuous belt is generated, providing that the orbital inclination θ_0 (defined here as the acute angle between the orbit plane and the equator) is not greater than the landing site latitude ψ plus the cross-range capability expressed in the great circle arc length l nor less than the landing site latitude minus the cross-range capability. If the latter limit is exceeded, the latitude coverage becomes zero, and return to the latitude of the landing

Received October 4, 1963; revision received May 22, 1964.
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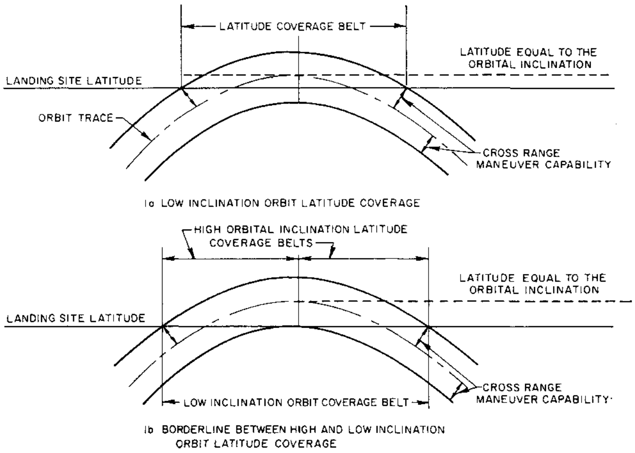

Fig. 1 Modes of latitude coverage.

site becomes impossible. The first limit is depicted in Fig. 1b. It is noted that an increase in orbital inclination would generate two separate latitude coverage belts during an orbital period. The latitude coverage corresponding to the two belts is shown in Fig. 2 and will be referred to as the high inclination mode of coverage. The one-belt mode of coverage will be referred to as the low-inclination mode of coverage.

The procedure for determining the magnitude of latitude coverage for the low-inclination mode will be discussed first. Because of earth rotation, it is convenient to consider the projection of the cross-range arc on a nonrotating reference sphere that is fixed with respect to the orbit plane. Neglecting the nodal regression rate of the orbit, the reference sphere corresponds to a spherical nonrotating earth. The latitude coverage can be conveniently determined on the reference sphere. The corresponding coverage on the rotating earth may then be determined by accounting for the relative motion between the earth and the nonrotating reference sphere during the generation of the latitude coverage belt. The limiting points of the latitude coverage belt on the reference sphere are defined by the condition that the end of the arc length representing the cross-range capability l, which extends perpendicularly from the orbit trace, is in contact with the landing site latitude (see Fig. 1a). The distance measured along the landing site latitude ΔL_c from one boundary of the latitude-coverage belt to the center of the belt may be determined as a function of the landing site latitude ψ, orbital inclination θ_0, and cross-range l as follows:

$$\cos\Delta L_c = (\tan\psi/\tan\theta_0) - [\sin l/\sin\theta_0 \cos\psi] \quad (1)$$

An arc length X_c measured along the orbit trace upon the reference sphere between the orbital position where a latitude-coverage belt boundary is established and a point on the orbit trace 90° from the ascending node may be determined from the following relation:

$$\cos X_c = (1/\cos l)[\sin\theta_0 \sin\psi + \cos\psi \cos\theta_0 \cos\Delta L_c] \quad (2)$$

The difference in the latitude coverage as represented on the reference sphere and on a rotating earth is established by the amount of earth rotation that occurs while the orbiting vehicle moves through a range angle equal to $2X_c$. Assuming a circular orbit, the distance the earth rotates is $\omega T(2X_c/360°)$, where T is the period of the circular orbit and ω is the earth's rotation rate. If the vehicle is orbiting in an easterly direction ($0 \leq$ orbital inclination $i < 90°$), the latitude coverage is reduced by $(\omega T/360°)2X_c$. If the vehicle is orbiting in a westerly direction ($90 \leq i \leq 180$), the term adds to the latitude coverage. Assigning a value of $+1$ to the constant A for the latter of the preceding situations and a value of -1 for the former situation, we have the low-orbital inclination latitude coverage L_L as follows:

$$L_L = 2\Delta L_c + 2A(\omega T/360°)X_c \quad (3)$$

The technique for determining the latitude coverage via the high-inclination orbit mode will now be described. With this mode two separate belts of latitude coverage of similar geometry are generated and are of equal magnitude (see Fig. 2). The latitude coverage of a single belt (two per orbit) designated L_H may be determined in a manner similar to the low inclination mode. Referring to Fig. 2, L_H may be considered to be composed of two components L_a and L_b that represent the coverage on either side of the orbit trace. The corresponding latitude coverages upon the reference sphere (nonrotating earth) are ΔL_a and ΔL_b and may be determined by the following relationships:

$$\cos\Delta L_a \sin\psi \cos y + \sin\Delta L_a \sin y = \sin\psi \cos y + (\sin l/\cos\psi) \quad (4)$$

and

$$\cos\Delta L_b \sin\psi \cos y - \sin\Delta L_b \sin y = \sin\psi \cos y - (\sin l/\cos\psi) \quad (5)$$

where

$$\cos y = \cos\theta_0/\cos\psi \quad (6)$$

The great circle arc lengths X_a and X_b (indicated in Fig. 2) along the orbit traces' projection upon the reference sphere are similar to X_c for the low orbital inclination mode and can be determined as follows:

$$\cos X_a = [1 - \cos^2\psi(1 - \cos\Delta L_a)]/\cos l \quad (7)$$

$$\cos|X_b| = [1 - (\cos^2\psi)(1 - \cos\Delta L_b)]/\cos l \quad (8)$$

Equation (8) determines only the absolute magnitude of X_b. The sign on X_b is positive if the arc X_b extends to the south of the landing site latitude (which is the case except for near polar orbits) or negative when the arc lies to the north. It may be determined by the criterion

$$X_b/|X_b| = \pm 1 \text{ when } z \gtrless y \quad (9)$$

where

$$\sin^2 z = (\cos^2\psi/\sin^2 l) - [(\cos l - \sin^2\psi)^2/(\cos^2\psi \sin^2 l)] \quad (10)$$

The latitude coverage upon the rotating earth may be determined by accounting for the earth's rotation in a similar manner as was done for the low inclination mode. Thus,

$$L_H = L_a + L_b = [\Delta L_a + A(X_a/360)\omega T] + [\Delta L_b + A(X_b/360)\omega T] \quad (11)$$

The terms involving ω have no effect upon the magnitude of the latitude coverage for polar orbits and have the greatest effect for the boundary case between the high and low-orbital inclination modes of coverage. The effect of the earth's rotation terms on the latitude coverage generally amounts to less than ten percent. Reference 3 considered a third set of terms which approximated the down-range maneuver potential, and it was found to have a negligible effect upon latitude coverage for near earth orbits.

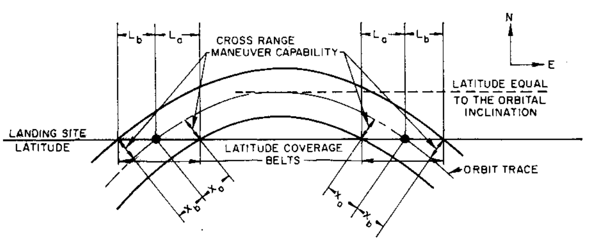

Fig. 2 High inclination orbit mode of latitude coverage.

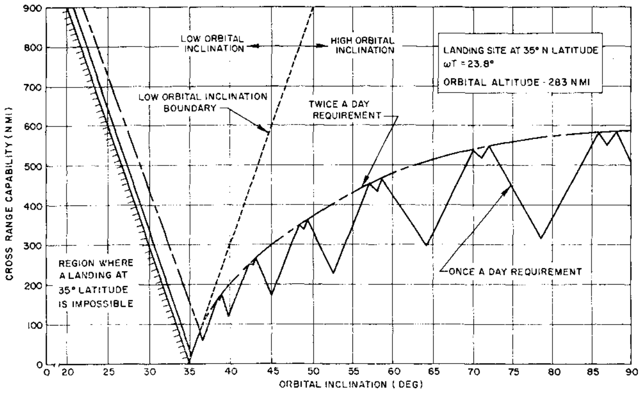

Fig. 3 Cross-range coverage to insure one and two landing opportunities each day.

An application of the latitude coverage thus determined is shown in Fig. 3. The curves given in Fig. 3 represent the cross range required to assure at least one and two landing opportunities each day, respectively, for a landing site located at a latitude of 35° (Edwards Air Force Base) as a function of orbital inclination. Generation of such curves requires the determination of the orbit trace spacing, at the landing site latitude, produced within the time period in which the landing opportunities are to be assured. Detailed procedures for this step are outlined in Ref. 3.

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Lunar Landing Guidance Using Cross-Product Steering

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A TERMINAL guidance technique is presented which insures a soft-landing at a preselected site on the lunar surface. The technique employs two feedback guidance channels to achieve continuous thrust vector control throughout the powered portion of the descent. One channel nullifies an error generated by application of a form of cross-product steering, thereby insuring passage of the vehicle trajectory through the desired landing site with a small horizontal velocity component (less than 2 fps) to avoid tipping. The other channel nullifies a second error signal, insuring a terminal value of vertical velocity which is within the bound (15 fps) required for a soft-landing. The simulation results presented are necessarily preliminary but provide a quantitative demonstration of feasibility.

Received January 31, 1964; revision received April 6, 1964.
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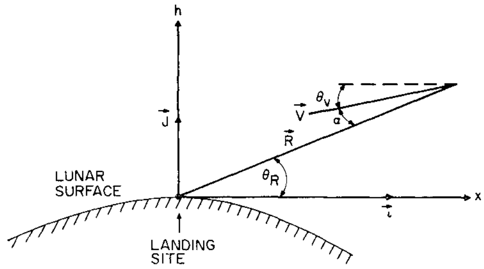

Fig. 1 Landing geometry.

Cross-Product Steering: Basic Relations

In order to arrive at a set of guidance equations having the simplest po sible form, it is assumed that landing occurs in a Cartesian coordinate system, with origin of the x-h plane at the desired landing site—an assumption that is justified for small angular travel during powered descent and that becomes more accurate as the landing site is approached.

The concept of cross-product steering for the planar case is illustrated by Fig. 1, which shows that the spacecraft will be forced to pass through the landing site if its velocity vector is aligned to the range vector, i.e., if $\tan\theta_R = \tan\theta_v$, which can be insured by nullifying the error

$$\epsilon_p = h\dot{x} - x\dot{h} \tag{1}$$

Forcing ϵ_p to zero is equivalent to forcing the vector cross product of **V** and **R** to zero

$$\mathbf{V} \times \mathbf{R} = VR \sin\alpha\cdot\mathbf{k} = \langle\dot{x}\mathbf{i} + \dot{h}\mathbf{j}\rangle \times \langle x\mathbf{i} + h\mathbf{j}\rangle = \langle h\dot{x} - x\dot{h}\rangle\cdot\mathbf{k} \tag{2}$$

Notice that if ϵ_p is maintained at zero, the nominal vehicle trajectory is a straight line. Equation (2) can be modified to give alternate preferred trajectories that allow the landing site to be approached along a more vertical path; this approach is more desirable if the vehicle is expected to hover before landing. The modification is achieved by setting

$$\beta \tan\theta_R = \tan\theta_v \qquad \beta = \text{const} \tag{3}$$

so that

$$\epsilon_p = \beta h\dot{x} - x\dot{h} \tag{4}$$

If this $\epsilon_p = 0$ throughout the descent, the trajectory is given by

$$h = (h_0/x_0{}^\beta)x^\beta = Kx^\beta \tag{5}$$

where h_0 and x_0 are the initial altitude and position. For fixed initia' position, and $0 < \beta \leq 1$, a family of trajectories is obtained (Fig. 2a). Alternately, if β is fixed but initial conditions are varied, a different family is generated (Fig. 2b). For each case, whenever $0 < \beta < 1$, the trajectories have the property that $\dot{x} \rightarrow 0$ as $x \rightarrow 0$.

Nullification of ϵ_p insures passage of the spacecraft trajectory through the desired landing site but does not simultaneously insure that a soft landing will be achieved. That

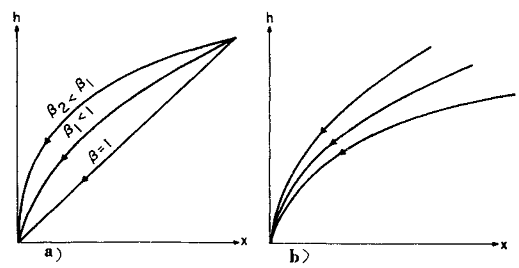

Fig. 2 Nominal ideal trajectories for $\epsilon_p = 0$: *a*) for fixed h_0x_0, variable β; *b*) for fixed β, variable h_0x_0.